

Saturation and geometrical scaling in small systems*

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Saturation and geometrical scaling (GS) of gluon distributions are a consequence of the non-linear evolution equations of QCD. We argue that in pp GS holds for the inelastic cross-section rather than for the multiplicity distributions. We also discuss possible fluctuations of the proton saturation scale in pA collisions at the LHC.

PACS numbers: 13.85.Ni, 12.38.Lg

At the eQCD meetings in 2013 and 2015 [1, 2] we have discussed the emergence of geometrical scaling for $F_2(x)/Q^2$ [3] in deep inelastic scattering (DIS) [4], and for charged particle multiplicity distributions in proton-proton collisions [5], and in heavy ion collisions (HI) [6]. Here, after a short reminder, we recall recent analysis [7] of ALICE pp data [8], and discuss a hypothesis that the saturation scale may fluctuate in the proton [9] on the example of the pA scattering as measured by ALICE [10] at the LHC.

The cross-section for not too hard gluon production in pp collisions can be described in the k_T -factorization approach by the formula [11]:

$$\frac{d\sigma}{dyd^2p_T} = \frac{3\pi\alpha_s}{2} \frac{Q_s^2(x)}{p_T^2} \int \frac{d^2\vec{k}_T}{Q_s^2(x)} \varphi_p\left(\vec{k}_T^2/Q_s^2(x)\right) \varphi_p\left((\vec{k}-\vec{p})_T^2/Q_s^2(x)\right) \quad (1)$$

where φ_p denotes the unintegrated gluon distribution that in principle depends on two variables $\varphi_p = \varphi_p(k_T^2, x)$. In Eq.(1) we have assumed that produced gluons are in the mid rapidity region ($y \simeq 0$), hence both Bjorken x 's of colliding gluons are equal $x_1 \simeq x_2$ (denoted in the following as x). Note that unintegrated gluon densities have dimension of transverse area. This

* Presented at *Excited QCD*, Costa da Caparica, Portugal, March 6 – 12, 2016.

is at best seen from the very simple parametrization proposed by Kharzeev and Levin [12] in the context of HI collisions:

$$\varphi_p(k_T^2) = S_\perp \begin{cases} 1 & \text{for } k_T^2 < Q_s^2 \\ k_T^2/Q_s^2 & \text{for } Q_s^2 < k_T^2 \end{cases} \quad (2)$$

or by Golec-Biernat and Wüsthoff in the context of DIS [13]:

$$\varphi_p(k_T^2) = S_\perp \frac{3}{4\pi^2} \frac{k_T^2}{Q_s^2} \exp(-k_T^2/Q_s^2). \quad (3)$$

In the case of DIS $S_\perp = \sigma_0$ is the dipole-proton cross-section for large dipoles and in (2) S_\perp is the transverse size of an overlap of two large nuclei for a given centrality class. In both cases one can assume that S_\perp is energy independent (or weakly dependent). Another feature of (2) and (3) is that $\varphi_p(k_T^2, x) = \varphi_p(k_T^2/Q_s^2(x))$ where $Q_s^2(x)$ is the *saturation momentum* that takes the following form $Q_s^2(x) = Q_0^2(x/x_0)^{-\lambda}$ motivated by the traveling wave solutions [14] of the non-linear Balitski-Kovchegov evolution equations [15]. In that case $d^2\vec{k}_T$ integration in (1) leads to

$$\frac{d\sigma}{dyd^2p_T} = S_\perp^2 \mathcal{F}(\tau) \quad (4)$$

where $\tau = p_T^2/Q_s^2(x)$ is a scaling variable and $\mathcal{F}(\tau)$ is a function related to the integral of φ_p 's. We shall follow here the parton-hadron duality [16], assuming that the charged particle spectra are on the average identical to the gluon spectra. Equation (4) has the property of GS if S_\perp is energy independent. In this case the entire energy dependence is taken care of by the energy dependence of τ .

In order to test relation (4) we shall use the fact that for mid-rapidity

$$\tau = p_T^2/Q_s^2(x) = p_T^2/Q_0^2 (p_T/(x_0 W))^\lambda \quad (5)$$

where $W = \sqrt{s}$, x_0 and Q_0^2 are constants that are irrelevant for the present analysis. We take $Q_0^2 = 1 \text{ GeV}^2/c$, $x_0 = 10^{-3}$. The only relevant parameter is λ . In Fig. 1 we plot ALICE pp data [8] in terms of p_T (left panel) and in terms of scaling variable τ (right panel) for $\lambda = 0.32$. We see that three different curves from the left panel in Fig. 1 overlap over some region if plotted in terms of the scaling variable τ . The exponent for which this happens over the largest interval of τ is $\lambda = 0.32$ [7], which is the value compatible with our model independent analysis of the DIS data [4].

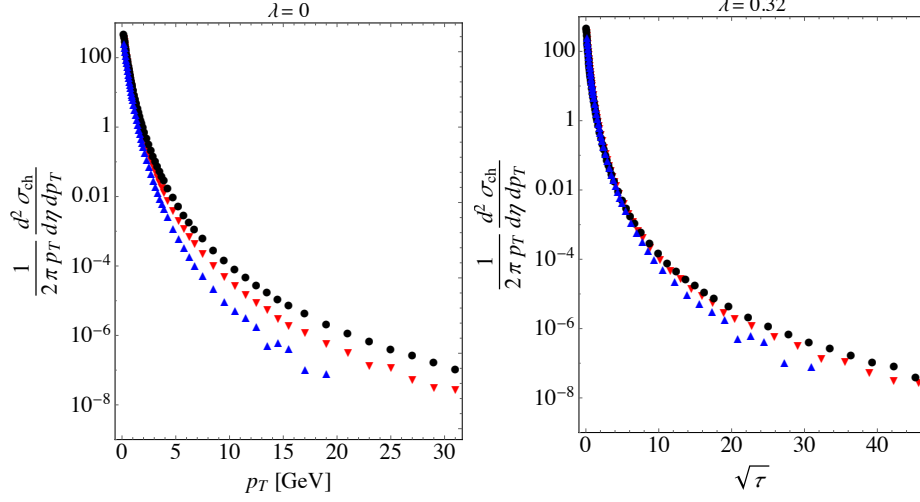


Fig. 1. Data for pp scattering from ALICE [8] plotted in terms of p_T and $\sqrt{\tau}$. Full (black) circles correspond to $W = 7$ TeV, down (red) triangles to 2.76 TeV and up (blue) triangles to 0.9 TeV.

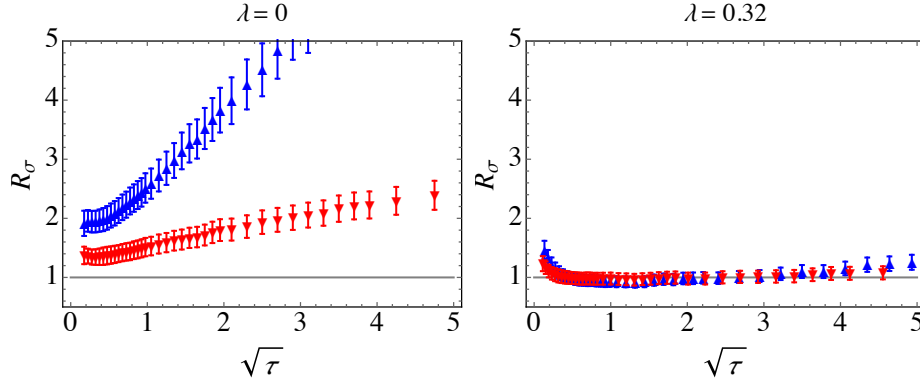


Fig. 2. Ratios of the cross-sections at 7/2.76 TeV – down (red) triangles and 7/0.9 TeV – up (blue) triangles, for $\lambda = 0$ (left) and 0.32 (right)

In order to illustrate the method of adjusting λ , we plot in Fig. 2 ratios of the cross-sections at 7 TeV to 2.76 and 0.9 TeV. Approximate equality of both ratios close to unity for $\lambda = 0.32$ is the sign of GS for p_T up to 4.25 GeV/c [7].

It has been argued previously that GS should hold for multiplicities, rather than for the cross-sections. This would be true if the relation between the two was energy independent. This may be the case in HI or pA collisions where we trigger on some S_\perp by selecting the centrality classes with given

number of participants, but it is not true in the case of the inelastic pp scattering:

$$\frac{dN}{dyd^2p_T} = \frac{1}{\sigma^{\text{MB}}(W)} \frac{d\sigma}{dyd^2p_T} = \frac{S_\perp^2}{\sigma^{\text{MB}}(W)} \mathcal{F}(\tau) \quad (6)$$

where the minimum bias cross-section $\sigma^{\text{MB}}(W) \neq S_\perp$ is energy-dependent. Repeating the procedure of constructing the ratios of the multiplicities rather than of the cross-sections, we find the best scaling for $\lambda = 0.22 \div 0.24$ [7]. This is illustrated in Fig. 3 where the left panel is just an enlarged version of the right plot of Fig. 2, whereas the right panel corresponds to the ratios of the multiplicities for $\lambda = 0.22$. We see that indeed multiplicity scaling is achieved for smaller λ , but – at the same time – the scaling is of worse quality than for the cross-sections and holds over a smaller interval of τ .

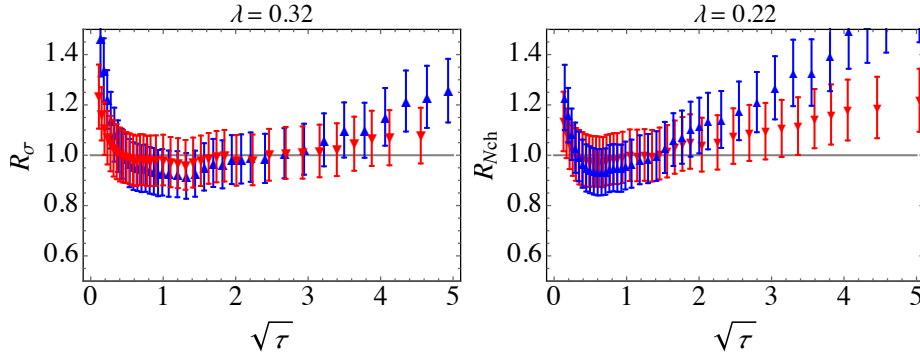


Fig. 3. Ratios of cross-sections (left) for $\lambda = 0.32$ and multiplicities (right) for $\lambda = 0.22$. For the meaning of symbols see Fig. 2

In the case of two different systems, like in the pA scattering and/or $y \neq 0$, formula (1) contains two different distributions $\varphi_{p,A}$ characterized by two different saturation scales $Q_{p,A}(k_T^2/s, \pm y)$. With simple parametrization (2) and assuming constant S_\perp corresponding to the definite centrality class, one arrives at a very simple formula for charged particle multiplicity [12]:

$$\frac{dN_{\text{ch}}}{dy} = S_\perp Q_p^2 \left(2 + \ln \frac{Q_A^2}{Q_p^2} \right). \quad (7)$$

Formula (7) predicts both energy and rapidity dependence and also N_{part} dependence of multiplicities through the dependence of the saturation scales

upon these quantities [12] (apart from S_\perp dependence on N_{part}):

$$\begin{aligned} Q_p^2(W, y) &= Q_0^2 \left(\frac{W}{W_0} \right)^\lambda \exp(\lambda y), \\ Q_A^2(W, y) &= Q_0^2 N_{\text{part}} \left(\frac{W}{W_0} \right)^\lambda \exp(-\lambda y) \end{aligned} \quad (8)$$

where we take $\lambda = 0.32$ as in DIS [4] and pp [7].

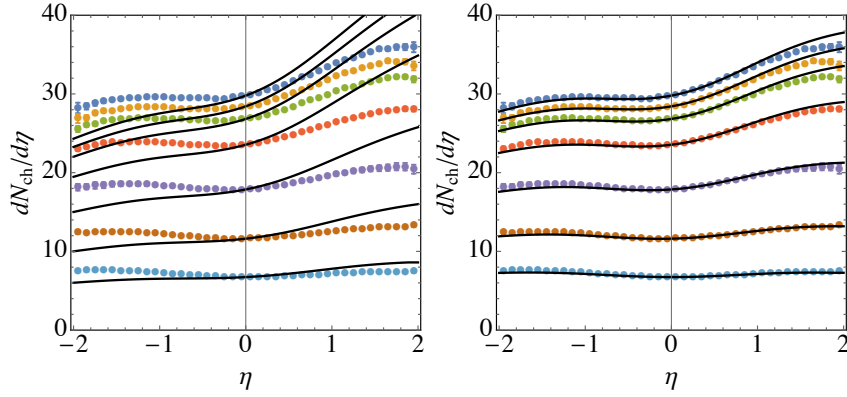


Fig. 4. Multiplicity spectra from Ref. [10] compared with the prediction of Eq. (7) without (left) and with fluctuations (right). For the meaning of symbols see Ref. [9]. Normalization of theoretical predictions has been fitted and is given by Eq. (10).

It has been shown in Ref. [9] that these simple formulae fail to describe recent proton-Pb LHC data [10]. To resolve this issue we have proposed to take into account possible fluctuations of the saturation scale in the proton according to the log-Gaussian distribution introduced in Ref. [17]

$$P(\rho) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left(-\frac{(\ln Q_s^2/Q_0^2 - \ln Q_p^2/Q_0^2)^2}{2\sigma^2} \right). \quad (9)$$

Here Q_s^2 is the proton saturation momentum fluctuating around its logarithmic average denoted as $\ln Q_p^2$ (with Q_0^2 being an arbitrary momentum scale, which cancels out in (9)) and σ is the fluctuation width, which we assume to be y independent (although it may in principle depend on W). Taking into account fluctuations (9) and the transformation from y to pseudorapidity η [9], we have been able to describe the multiplicity distributions adjusting the normalization in Eq. (7) for each centrality class. In Fig. 4 we show the results for the ALICE data for centrality class determination by the ZNA method ($N_{\text{part}} = N_{\text{coll}}^{\text{Pb-side}} + 1$ from Table 7 in Ref. [10], whereas in Ref. [9]

we have used V0A centrality determination). As in Ref. [9] we have to take rather large $\sigma \sim 1.55$ to describe the data. The normalization S_{\perp} has been fitted to the data by means of the logarithmic parametrization:

$$S_{\perp} = (0.88 + 0.47 \ln N_{\text{part}})^2. \quad (10)$$

To summarize: We have presented new developments in the studies of GS for small systems, *i.e.* for pp and pA collisions. We have shown that a good quality scaling in pp is achieved for the inelastic cross-sections rather than for the multiplicities. In the case of pA collisions we have reported on a recent proposal to include the fluctuations of the saturation scale of the proton in order to describe recent data on multiplicity distributions $dN_{\text{ch}}/d\eta$ for different centrality classes.

This work was supported by the Polish NCN grant 2014/13/B/ST2/02486.

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